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A Behavioral Theory of Interest Rate Formation

by

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Introduction. This paper develops and marshals empirical support for a theory of the nominal, short-term, risk-free interest rate. Unlike most approaches to the problem, the formulation presented here does not assume equality of supply and demand for securities. Rather, the formulation is a mechanism which would help to bring supply and demand into balance in a larger macroeconomic model. Empirical support is offered at two levels of aggregation: Empirical studies of individual human decision making are used to suggest an appropriate structure; the structure is then estimated using macro-economic data.

In the proposed formulation the risk free interest rate moves in response to liquidity pressures experienced principally by intermediaries. Liquidity pressures cause movements in the interest rate relative to the "underlying interest rate", a construct which represents the interest rate environment to which people have become accustomed.

The formulation has grown from work on the M.I.T. System Dynamics National Model (for a description of the modeling project see Forrester 1984 and 1979). The National Model is a simulation model of an industrial economy. It possesses two production sectors, a household sector, a financial sector, and a government sector. The Model is intended to aid in understanding the major behavior modes of industrialized economies including the business cycle, the Kuznets cycle, the economic long wave, inflation and deflation, and economic growth. (For discussion of the several major economic behavior modes see Schumpeter 1944; Volker 1978; and Forrester 1976, 1982).

In some respects the National Model represents a departure from prior economic modeling. The nature of this departure gives rise to the need for a new interest rate formulation. Three characteristics are particularly important in this regard. First, the National Model does not permit exogenous time-dependent inputs¹; consequently, exogenous interest rates could not be used as explanatory variables in the interest rate formulation. Using exogenous interest rates begs the question of how interest rates are formed. Second, the National Model unfolds in continuous time, and, therefore, the use of simultaneous equations is less appropriate than in most other economic

1. More precisely, the only exogenous inputs are noise and idealized test inputs designed to disturb the system and elicit its dynamic response.

models. Finally, model construction has emphasized the portrayal of decision making heuristics more than the derivation or application of optimizing decision rules.

The emphasis on the nominal, short-term, risk-free interest rate is one of convention. The important simplifying strategy in a macro-model examining major movements in the economy (as opposed to fine movements in interest rates) is to form one basic interest rate with strict regard to the rigor imposed by the requirements of behavioral modeling. Other rates can then be derived from this one as needed. Any rate can be regarded as basic; other rates may then be calculated by adjustments up or down depending upon whether those rates are more or less risky than the basic rate².

As the basic rate, I have chosen the nominal, short-term, risk-free interest rate. This choice is in broad keeping with economic tradition, although many theorists would focus on the real, short-term, risk-free interest rate. In the United States, however, there are no instruments carrying the real rate, and consequently representing demand and supply pressures in a behavioral model might become problematic were the real rate used. The nominal risk-free, short-term interest rate, on the other hand, corresponds closely to the interest rate on a federal government note. The note is virtually risk-free in terms of default risk because the government has the ability to print money to pay its debt.³

The first section following considers the the more traditional equilibrium approach to interest rate formation including Walras' tâtonnement adjustment process. The next section presents an alternative adjustment process based upon a modification to Walras' auctioneer. The "auctioneer" of the new formulation permits trades out of equilibrium and is willing to buy and sell for his own account. The third section of this paper discusses the behavior of the new formulation and summarizes statistical tests.

1. Simultaneous Equations and Tâtonnement

Simultaneous Equations. The theory of interest rate formation developed in this essay is fundamentally a supply and demand formulation. The formulation, however, is distinguishable from supply-demand formulations in most other econometric work. In such work interest rates are

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2. Translations between short and long rates would also involve expectations regarding changes in the short or long rates and, perhaps, risk premia for different maturity habitats (Modigliani and Shiller 1973, Modigliani and Sutch 1966).
 3. There have been instances in which governments have defaulted on debt denominated in local currency, although such instances are not common. The default risk on the debt of the industrialized nations is extremely small.

chosen to equate desired supply and desired demand (see for example, Friedman 1980a, 1980b, 1977; Friedman and Roley 1980, Modigliani, Rasche, and Cooper 1970, Hendershott 1977, Bosworth and Duesenberry 1973). More specifically, the desired (primary) purchases of a security by lenders and the desired issues of that security by borrowers are specified as functions of the interest rate on that security. The condition that desired purchases (demand) equal desired issues (supply) is then imposed. With this condition the desired amount of securities and the interest rate can be determined.⁴

For example, if $g(i,z)$ is a function determining desired loans DL (or securities purchases) and $f(i,z)$ a function determining desired borrowing DB (or issues), where "i" is the interest rate and "z" is exogenous factors; a system of three simultaneous equations may be written:

$$DL = g(i,z) \quad (1)$$

$$DB = f(i,z) \quad (2)$$

$$DL = DB. \quad (3)$$

Equations (1) and (2) may be substituted into (3) to yield

$$g(i,z)=f(i,z) \quad (4)$$

which may be solved for a unique interest rate i under suitable restrictions on g and f . The interest rate so discovered may then be substituted into either (1) or (2) to uncover the amount demanded and supplied.

It might be noted that the simultaneous equations approach is an equilibrium approach. In equilibrium desired quantities equal actual quantities.⁵ The desires of sellers and buyers can both be fulfilled only if the quantities they each have in mind are the same, otherwise one of the groups will find itself short of transacting partners. The simultaneous equations approach requires that

4. Modigliani, Rasche and Cooper (1970) equate the public's demand for demand deposits to the supply offered by banks. The IS-LM approach makes use of simultaneous equalities between goods supplied and goods demanded (IS) and between money demanded and money supplied (LM) to obtain equilibrium figures for GNP and interest rates (Dornbusch and Fischer 1981, Ch. 4, eq. 6a and 12a).

5. More precisely, desired quantities equal actual quantities in a stress-free equilibrium. Most generally, an equilibrium might be considered as a condition in which all states (integrations) of a system are constant or are growing at a constant rate. Such a condition can result when desires are not fulfilled but where opposing forces in the system conspire to prevent further movement of actual quantities toward desired. This is termed a "stressed" equilibrium. There is a pervasive belief among economists (including economists working in the system dynamics tradition) that actors in the economic system can analyze a stressed equilibrium and take action to achieve a more satisfactory equilibrium.

sellers' desired quantities always equal buyers' desired quantities (equation 3). Hence, the simultaneous equations approach carries the assumption that financial markets are always in equilibrium.

Real-life interest rates are not literally determined by the solution of simultaneous equations. This conclusion results both from observation (people do not *appear* to solve simultaneous equations) and from a recognition of the informational and cognitive limitations faced by decision makers: No individual or group of individuals could possibly know all the functions of all the players in the financial markets. And, even if all functions were known, the solution of the set of simultaneous equations resulting therefrom would be prohibitively expensive, if possible to achieve at all.

Since the process of solving simultaneous equations does not literally describe the real world process of interest rate formation, use of simultaneous equations in models of interest rate determination must rest upon the assumption that a process exists in the real world which quickly equates the desired demand to the desired supply of securities (Cf. Eberlein 1984, p. 183 ff.), . In a modeling context, "quick" means quick in relation to other processes represented in the larger model within which the interest rate formulation is embedded. In an empirical context, "quick" means quick relative to the frequency of observations.

The use of simultaneous equations in the present context seems less appropriate than in the work of Friedman and others cited above for two reasons. First, the previously cited work was intended for quarterly models, while the formulation presented in these pages is intended for a continuous-time model. Second, the formulation presented below is intended to account for the general level of interest rates, not the differences between rates on different securities. Since most of the previously cited work includes, as explanatory variables, rates on other securities; that work may be considered as accounting for interest rate differentials which may move faster than the entire structure of interest rates taken as a whole.⁶

The Adjustment Process. The present work does not assume simultaneities, but focusses on the process which moves interest rates toward equilibrium. Walras (1954) appropriately called this process "a groping" or tâtonnement process. In Walras' formulation, an "auctioneer"⁷ calls out an interest rate, participants make offers to lend and borrow at this interest

6. Modigliani, Rasche and Cooper (1970) do not use exogenous interest rates as explanatory variables.

7. The term "auctioneer" is common. "Master of Ceremonies" would better describe the intended function.

rate. If desired loans do not match desired borrowing, the auctioneer calls out a new interest rate. He continues in this fashion until offers to borrow equal offers to lend, at which point he permits the participants to transact.

While Walras' process is considered even by those who use it as "a fairly extreme idealization of the mechanism by which prices are determined" (Malinvaud 1972, 140), it is well to note that it is actually only a bit less unwieldy than the solution of simultaneous equations. Instead of requiring massive amounts of information and computational power, this process requires the massive and patient cooperation of institutions and the public at large. The process is really little more than a hypothetical computational technique for the solution of a set of simultaneous equations (Goodwin 1951).

2. An Alternative Adjustment Process.

Realism and Computational Ease. The lack of realism in Walras' tâtonnement may be identified in at least two linked ways. First, transactions cannot take place before an equilibrium price is determined, whereas on real-world commodity and stock exchanges "without exception contracts are made at each of the prices called." (Malinvaud p. 139). Second, the auctioneer, although a kind of intermediary, is not permitted to take a position in the market, that is, he is not permitted to buy and sell for his own account; unlike his real-life counterparts. Realism may be improved, therefore, by permitting transactions out of equilibrium and permitting the intermediary to buy and sell for his own account.

A straightforward improvement in realism may be achieved by replacing Walras' auctioneer with an aggregated financial intermediary. The simplification involved in taking this step is quite the reverse of that made by Walras: The intermediary, rather than being a party to no transactions, can be assumed to be a party to every transaction. Such an arrangement decreases the computational effort required by the auctioneer and points the way to a realistic and practical way of representing interest-rate determination in a dynamic, continuous-time macro-economic model.

If the intermediary stands ready to borrow and lend, it will find the information necessary to set interest rates encapsulated in its inventory of securities or of money. The inventories integrate, and thereby summarize, the entire history of transactions. A large inventory of securities or a small inventory of money indicates borrowers have been selling securities to the intermediary at a rate greater than investors have been buying securities from it. When the intermediary's money balances are low, that is when it is illiquid, it should, and it will be motivated to, raise the

price at which it will buy or sell from its own account⁸. The converse holds when securities are low and liquid funds are high.

Assuming that intermediaries are prepared to buy and sell from their own accounts at a price (as long as their supplies last) implies a decoupling of quantities demanded and supplied. If intermediaries are willing to absorb differences between supply and demand, at least while their inventories last, there is no rigid requirement that desired supply equal desired demand in the non-intermediary sectors. Hence, the formulation suggested here is a disequilibrium formulation in the sense that desired supply does not have to equal desired demand.⁹

It is important to note that while the formulation does not assume equality between supply and demand, it does not preclude equality. Indeed, this formulation would be one of the mechanisms tending to bring a larger macro-economic model to an equilibrium in which desired demand for securities equals desired supplies.

It is interesting that the description of interest rate formation in terms of intermediaries is valid for the particular market in which the risk-free rate is set. In that market government securities dealers are the primary intermediary (often dealing with other intermediaries such as banks). In giving a first-hand description of the Federal Reserve's open market transactions on a particular day, Paul Meek (1978) writes: "...A last-minute contact with the traders indicates that banks and others are beginning to sell treasury bills to dealers in greater volume than buyers are taking from them. Dealers are raising the rates they are bidding for treasury bills...". I might add that dealers at this point have a low "inventory" of liquid funds.

The question remains as to how to formulate, in a way consistent with observation, the manner in which intermediaries determine a reasonable interest rate. In the modern interpretation of Walras (Samuelson's (1947, p. 270), Goodwin 1951, Negishi 1962) price movement (dp/dt) is proportional to the difference between supply and demand. This formulation possesses the virtue of simplicity and may be well suited to the analytical needs of stability theory, but it is not well grounded in empirical observation of how people make decisions and is, therefore, less well suited to the needs of a behavioral macro-economic model. In addition, the formulation assumes continuity in interest rate movements. Since prices are not physical accumulations, there is no

8. An intermediary may also restrict its lending activities.

9. A further way in which the formulation can be a disequilibrium formulation is that there is no requirement that sectors receive what they desire. This is consistent with the observed occurrence of credit crunches (Sinai 1976).

reason to make this assumption. It is desirable to create a formulation which does not assume continuity and which is firmly grounded in observation of human decision making.

An appropriate place to begin thinking about the way people choose rates is the literature on individual decision making. A large and growing body of such work suggests that the process termed "adjustment and anchoring" is a common strategy for arriving at judgments like interest rate determination (Hogarth 1980 p.47, Tversky and Kahneman 1974, Kleinmuntz 1985). In the adjustment and anchoring process people come to a judgment by adjusting a preexisting quantity (the anchor) to take account of currently available information.

The empirically observed adjustment and anchoring strategy is here hypothesized to undergird the interest rate setting process in the real world.¹⁰ More particularly, it is hypothesized that in the presence of supply and demand imbalance, summarized by the inventory positions of intermediaries¹¹, people choose a current interest rate by adjusting up or down an anchor here termed the "underlying interest rate". The underlying interest rate represents the interest rate environment to which people have become accustomed.

A mathematical formulation suitable for use in a simulation model involves clarifying three issues. First, it is necessary to consider the manner in which the current interest rate is adjusted above or below the anchor; second, the way people form the anchor itself must be considered; and, finally, the manner in which inventories are translated into an adjustment term must be described.

Multiplicative Adjustment. Kahneman and Tversky (1982, p.168) present results suggesting that a given difference in price is perceived as less important as the price increases. For example a five dollar difference in price on a \$15 item is subjectively more important than a five dollar difference on a \$125 item. This suggests that for a given state of liquidity the difference between the anchor and the risk-free rate should increase as rates increase in order for the psychological impact of the difference to remain the same. A functional form in which the effect of liquidity enters multiplicatively possesses this characteristic.

10. This discussion makes an aggregative leap. Hogarth (1980) and Tversky and Kahneman (1974) examined individual decision making. Here I am discussing an aggregation of decisions made by many people. As discussed below, this consideration suggests the need for empirical validation at a macro level.

11. It is possible to formulate this either on the basis of the inventory positions of intermediaries alone or on the basis of the inventory positions of both intermediaries and non-intermediaries. For realistic adjustment times in the non-intermediary sector, there is little dynamic difference since money-inventory imbalances are corrected quickly.

$$RFIR_t = UIR_t * ELR_t \quad (5)$$

$$ELR_t = f(\text{intermediaries' liquidity}_t) \quad (6)$$

Where RFIR - Risk Free Interest Rate
 UIR - Underlying Interest Rate
 ELR - Effect of Liquidity on the risk free Rate
 f A decreasing function

In equation 5 the effect of supply and demand enters multiplicatively, indicating that the difference between the risk free rate and the underlying rate depends upon the underlying rate. Hence, during periods of high interest rates, indicated by a high value of UIR (see below), the gap between UIR and RFIR will be greater than during periods of low rates for identical values of ELR.

The Underlying Interest Rate. The underlying rate, analogous to the "underlying inflation rate" or the "underlying unemployment rate", is that rate which people feel generally holds at the present time, abstracting from the effects of transitory pressures on the actual interest rate. The underlying interest rate is the reference condition to which people have become accustomed; it continually adjusts as interest rates change and people become accustomed to the changed environment. A weighted average with recent experience weighted most heavily would seem to be an appropriate mathematical representation of this concept. An exponential average possesses this property and is easily representable in a simulation model (Forrester 1961, p. 406-411). While a small modification will be considered in the section of this essay dealing with behavior and in appendix 2, for most purposes the underlying rate may be defined as:

$$\dot{UIR}_t = (RFIR_t - UIR_t)/TAUIR \quad (7)$$

Where UIR - Underlying Interest Rate
 RFIR - Risk Free Interest Rate
 TAUIR - Time to Adjust Underlying Interest Rate
 and the dot (•) represents a time derivative.

The speed of adjustment of UIR toward RFIR depends on the parameter TAUIR. The diagram below charts the adjustment path of UIR for a step increase in RFIR for several values of TAUIR.

rate will be the same in an equilibrium with constant nominal rates and no maturity preferences. Hence, a notion of a future stress-free (hence, equilibrium) short rate is very close to a notion of a normal long rate. Indeed, Modigliani and Sutch (1966) (M-S) suggest that Keynes's normal rate can be approximated by a weighted average of past experience where recent experience is weighted most heavily. An exponential average possesses this property; hence, (7) conforms to the M-S computational form of the normal rate.

Modigliani and Sutch also considered an extrapolative component in expectations of future rates. M-S, however, were concerned with interest rate expectations over the entire course of the future, rather than with the very long-run expectation of the future short rate. It seems possible that the extrapolative component reflects expectations of continuing pressures over the short and medium term. Such extrapolative terms may not be present in the expectation of the future short rate at such time when pressures have relaxed.

Modigliani and Shiller (1973) reconsidered the problem of interest rate expectations in the light of inflationary expectations. They concluded that interest rate expectations include a separable effect of inflation. Whether this would hold true for the underlying rate which connotes a rate to which people have become accustomed is less clear. In any event, the fact that the extrapolative component may be unimportant in forming the underlying rate suggests that a separate impact of inflation might be difficult to discern if expectations of inflation are also regressive (cf. Modigliani and Shiller, 20-21).

Effect of Liquidity. While the terminology of "liquidity" tends to restrict attention to money balances, it is well to note that the effect with which we are concerned can be viewed as a function either of the inventories of securities or the inventory of money. One can work with either inventory because an increase in one implies an equal decrease in the other.

Because the concern here is with movements common to rates on all securities, a potentially troublesome aggregation issue can be largely avoided by considering the inventory of money, since instruments used as money will be quite similar. Consequently, money inventories will be explicitly considered and the "liquidity" terminology is descriptive.

The simplest variable measuring the state of liquidity is the difference between actual and desired money holdings. However, this difference will be sensitive to the price level and institutional size, which one might expect to be unimportant in the determination of interest rates. Inventory discrepancies, therefore, should be measured relative to some base. Either desired inventories or actual inventories are prime candidates. Choosing inventories as the base yields the following expression where intermediaries' money inventories are termed "reserves":

$$ELR = f(RAR) \quad (8)$$

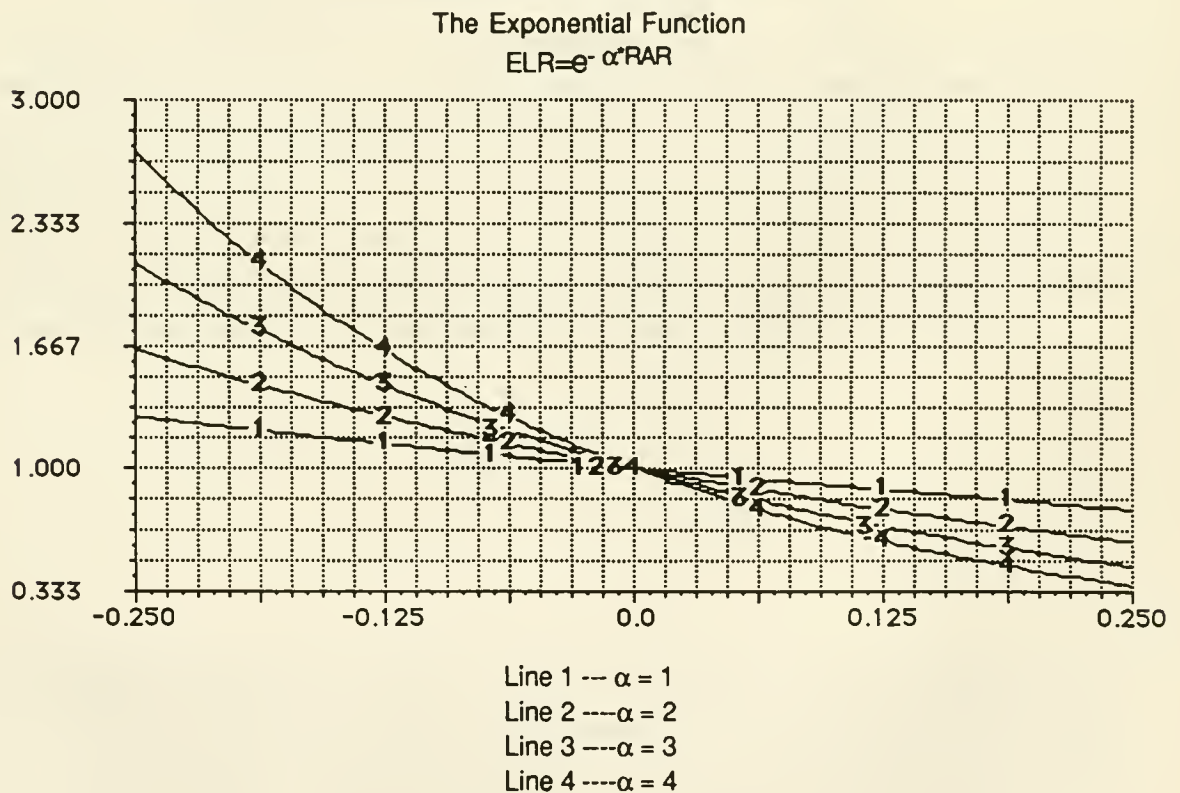
$$RAR = (R - DR) / R \quad (9)$$

Where: ELR -Effect of Liquidity on the risk free Rate
 RAR -Relative Available Reserves
 R -Reserves
 DR -Desired Reserves
 f -a decreasing function

The function f is not completely arbitrary. It must take the neutral value of 1 when RAR takes the value of 0, indicating reserves are equal to desired. It must be negatively sloped indicating that the interest rate will increase as intermediaries become less liquid. The function cannot drop below zero, because nominal rates are never negative. A flexible function which satisfies these properties and which will prove quite convenient later is the exponential function:

$$ELR = f(RAR) = e^{-\alpha \cdot RAR} \quad (10)$$

This function, for several values of α , appears below:



Again the estimation of parameters is discussed below. It is appropriate here to consider what reasonable values of α might be. One can gain a sense for reasonable values of α by

considering the relative available reserves of the banking system where, as discussed below, desired reserves are taken to be identical to required reserves. RAR went as low as -.25 during 1984 and as high as 0. Rates remained between 11% and 8%. Consequently, a value of α in the range of 1 or 2 would seem reasonable.

3. Behavior and Estimation.

Behavior. Equations 5,7,9, and 10 constitute a theory of interest rate formation. The equations are reproduced below for convenience.

$$RFIR_t = UIR_t * ELR_t \quad (11)$$

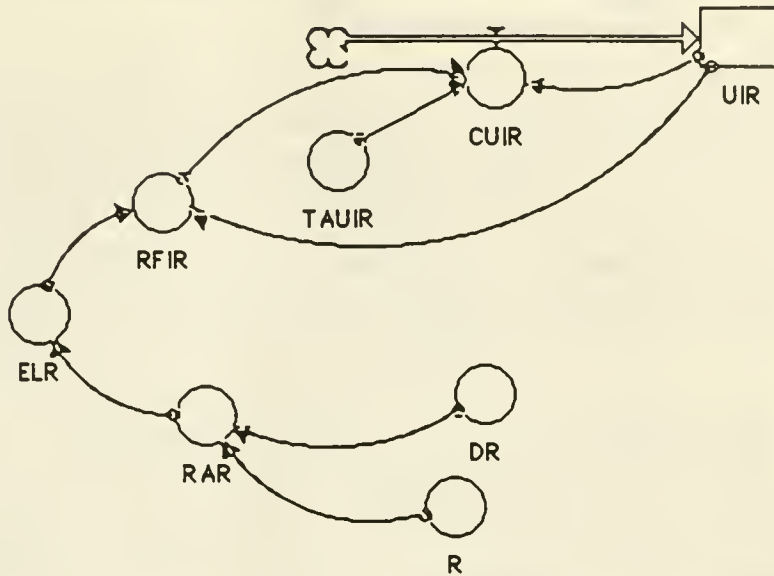
$$\dot{UIR}_t = (RFIR_t - UIR_t) / \tau_{UIR} \quad (12)$$

$$ELR_t = f(RAR_t) = e^{-\alpha * RAR_t} \quad (13)$$

$$RAR_t = (R_t - DR_t) / R_t \quad (14)$$

Where - RFIR - Risk Free Interest Rate
 UIR - Underlying Interest Rate
 ELR - Effect Liquidity on the risk free Rate
 τ_{UIR} - Time to Adjust Underlying Interest Rate
 R - Reserves
 DR - Desired Reserves

The stock and flow diagram¹² of the structure may be drawn as follows:



A sense for the dynamics implied by the above structure may be gained by considering a plot, shown below, from a simulation. In this simulation TAUIR (measured in years) and α are both set to 2, a reasonable value for each as discussed above. Initially UIR is 2% per year, and reserves are set equal to desired reserves, so RAR begins at its equilibrium value of zero. The structure is disturbed from equilibrium by a 10% step decrease in reserves; after some years reserves step up to their original (equilibrium) level. Plotted in the diagram below are the risk free interest rate RFIR, the underlying interest rate UIR, and relative available reserves RAR.

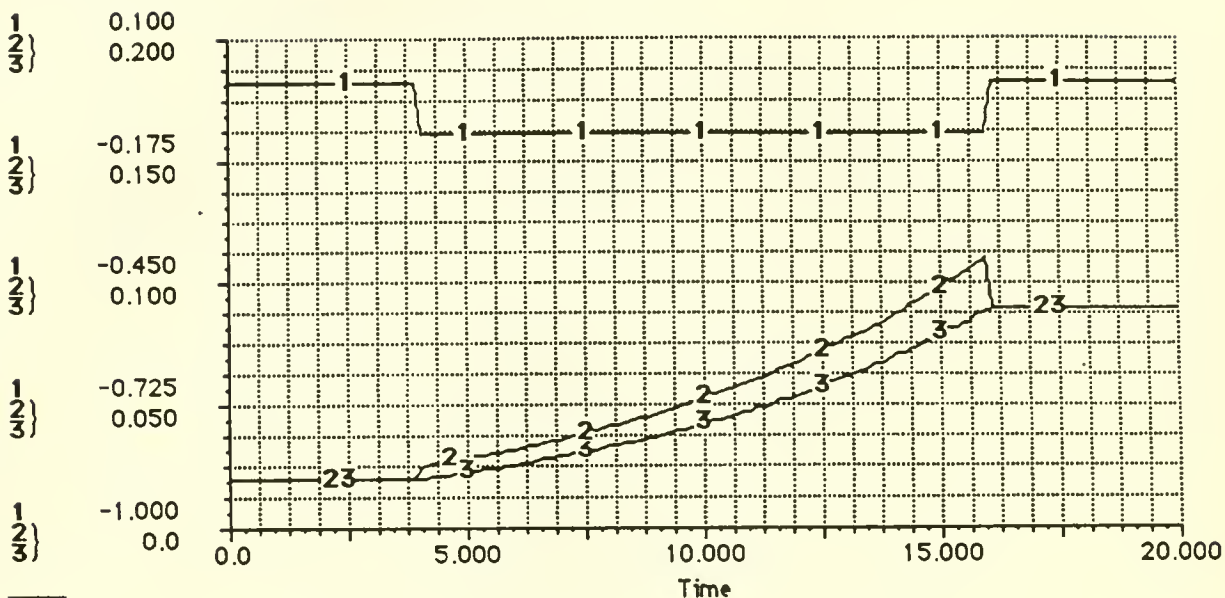
12. Stock and flow diagrams are in common use in system dynamics. They provide a close pictorial representation of a system of integral equations. The diagrams appear in several styles, the above diagram was created using STELLA® on a Macintosh computer. In the above diagram, the box represents an integration. The "Pipeline" arrow into the box represents a rate of flow which is controlled by the "valve" symbol termed CUIR (Change in UIR). CUIR, mathematically, is the time derivative of UIR. Circles represent constants or functions. Single-line arrows represent information connections and reveal the arguments of each function. For a good discussion of diagramming conventions in system dynamics see Richardson and Pugh (1981, chapters 1-2).

Interest Rate Formulation Simulated Behavior

1 RAR

2 RFIR

3 UIR



Initially the structure is in equilibrium, with the risk free rate and the underlying rate equal at 2%. Because desired reserves remain unchanged, the 10% decrease in reserves translates into a step down in RAR from 0 to -0.1 , as shown in the diagram. ELR immediately rises to about 1.2 and, as seen in the diagram, the risk free rate rises to about 120% of the underlying interest rate. The underlying rate now begins adapting to the new rate environment, that is it begins moving toward the risk free rate. As long as RAR remains at -0.1 , however, the adjustment to the risk free rate remains at 120% of the underlying rate. Consequently, the underlying rate moves toward an advancing target and, as long as reserves remain low, both the risk free rate and the underlying rate grow exponentially.

The step increase in reserves to its original level returns RAR to its neutral value of 0. This in turn, relieves pressures in the financial system and the risk-free rate returns to the underlying interest rate. With balance in supply and demand restored, there are no further pressures to generate movement in the rates. Had the structure been expanded to include a feedback effect involving desired money balances in the non-financial sector, pressures would have been generated which would have returned the risk-free rate and the underlying rate to their initial equilibrium values.

The analysis of the above behavior provides the logic for uncovering a flaw in the formulation. Just as the structure causes interest rates to increase exponentially in response to a maintained decrease in reserves, the structure will cause rates to decay exponentially in response to a maintained increase in reserves. Equation 11, however, indicates that once the underlying rate reaches zero, the risk free rate will also equal zero. Equation 12 indicates that if the two rates are identical, no further movement occurs in the structure. Consequently this structure carries the small risk that rates will get "stuck" at zero.¹³ The appendix to this essay contains a small modification to the structure developed above which removes the risk entirely. Since evidence of the small modification would be difficult, if not impossible, to pick up in the available time-series data, I proceed directly to a discussion of empirical validity.

Estimation. The structure developed above has been based upon empirical observation at the individual level, whereas it is intended to represent macro behavior. It is necessary now to present evidence that the formulation is consistent with macro-economic observation. Toward this end, the structure will be estimated econometrically, significant parameter estimates of reasonable magnitude and predicted sign will constitute evidence that the formulation is consistent with macro-economic data on interest rates and reserves.

The first step is to convert equations 11-14 into a form which can be estimated. A convenient way to do this is to solve the differential equation¹⁴. Substituting 11 into 12 and solving for UIR yields:

$$\ln(\text{UIR}_t) = \ln(\text{UIR}_{t_0}) + (1/\text{TAUIR}) \int_{t_0}^t (\text{ELR}_s - 1) ds \quad (15)$$

From (11) we know that:

$$\text{UIR}_t = \text{RFIR}_t / \text{ELR}_t. \quad (16)$$

Substituting 16 into 15 and rearranging yields

13. In a truly continuous-time simulation, UIR will never reach zero. However, since actual simulations must proceed by finite steps, UIR can hit zero in an actual simulation. Furthermore, the problem of getting "stuck" does not materialize suddenly; a UIR "very close" to zero will be "very close" to being stuck at that value.

14. It would also be possible to approximate the differential equation with a difference equation. As will be seen, the solution of the differential equation results in a "quasi" linear equation. The difference equation would be non-linear. Further, there is some evidence that the use of difference equations as an approximation to an underlying continuous-time model can cause problems (Richardson, 1981).

$$\ln(\text{RFIR}_t) = \ln(\text{UIR}_{t_0}) + \ln(\text{ELR}_t) + (1/\text{TAUIR}) \int_{t_0}^t (\text{ELR}_s - 1) ds \quad (17).$$

Substituting for ELR using 13 and reexpressing the integral produces:

$$\ln(\text{RFIR}_t) = \ln(\text{UIR}_{t_0}) - \alpha * \text{RAR}_t + (1/\text{TAUIR}) \left\{ \int_{t_0}^t e^{-\alpha * \text{RAR}_s} ds - (t - t_0) \right\} \quad (18)$$

or for estimating purposes:

$$\ln(\text{RFIR}_t) = k + z * \text{RAR}_t + h * \{ \text{CELR}_t - (t - t_0) \} \quad (19)$$

Where $z = (-\alpha)$, $h = (1/\text{TAUIR})$, and CELR is a month by month accumulation.

The integration in equation 18 may be approximated with a month-by-month accumulation CELR. The approximation would be exact if RAR were constant during the month, changing only in the instantaneous transition from one month to the next.

The entire equation may be estimated by the following method: Start with an estimate of α and form the month-by-month accumulation of the exponential CELR. Next, estimate h and the coefficient z using ordinary least squares. Form a new month-by-month accumulation CELR by adjusting the old estimate of α toward $(-z)$. Continue in this manner until the α used to form the accumulation CELR and $(-z)$ converge.

The question of what data to use in estimating 19 remains to be discussed. Data on the risk-free nominal interest rate poses no significant problem since time series on treasury bills are readily available (see appendix 1). Observation of RAR poses a greater problem. Data relating to depository institutions can be used with the assumption that the reserve positions of depository institutions are highly correlated with the reserve positions of other intermediaries. This assumption is discussed at greater length below.

In forming RAR for depository institutions, a problem exists. While information concerning the reserves of depository institutions is available (see appendix 1), data on desired reserves is not. Here, I will use required reserves (see appendix 1) as a proxy for desired reserves¹⁵. This seems conscionable since required reserves are likely the major determinant of desired reserves. Other factors, such as changing interest rate spreads (Cf. Modigliani, Rasche

15. The difficulty in deriving an estimable expression for free reserves is noted by Modigliani, Rasche, and Cooper (1970).

and Cooper (1970, especially eq. 3.11), fluctuations in the degree of Federal Reserve displeasure at borrowing from the discount window, the covariance between deposits and withdrawals, and the degree of risk in the lending portfolio, will result in what are likely to be minor variations in desired reserves away from the level of required reserves.

It is possible to consider the distortion introduced by using RAR of depository institutions instead of RAR of all intermediaries. It seems likely that the relative inventory positions of all intermediaries are highly correlated because financial instruments may be quickly traded between intermediaries. It is difficult to imagine a pool of excess liquidity obtaining for a significant period in one set of intermediaries while another class is illiquid. It must be true that the relative available reserves of different intermediaries are correlated. This means that¹⁶:

$$RAR_t = j \cdot RAR_t^b + e_t \quad (20)$$

where RAR^b is relative available reserves for depository institutions, e is a disturbance term and $j > 0$. This means that

$$ELR_t = e^{-\alpha \cdot RAR_t} \approx e^{-\alpha \cdot j \cdot RAR_t^b} \quad (21)$$

and the estimate $(-z)$, obtained from 19 is actually an estimate of $\alpha \cdot j$. Since theory calls for positive j and α , a positive estimate $(-z)$ would constitute evidence in favor of the theory.¹⁷

OLS estimation of equation 19, using data from January of 1959 to June of 1985¹⁸, produces the results summarized on line 1 in figure 1 below.

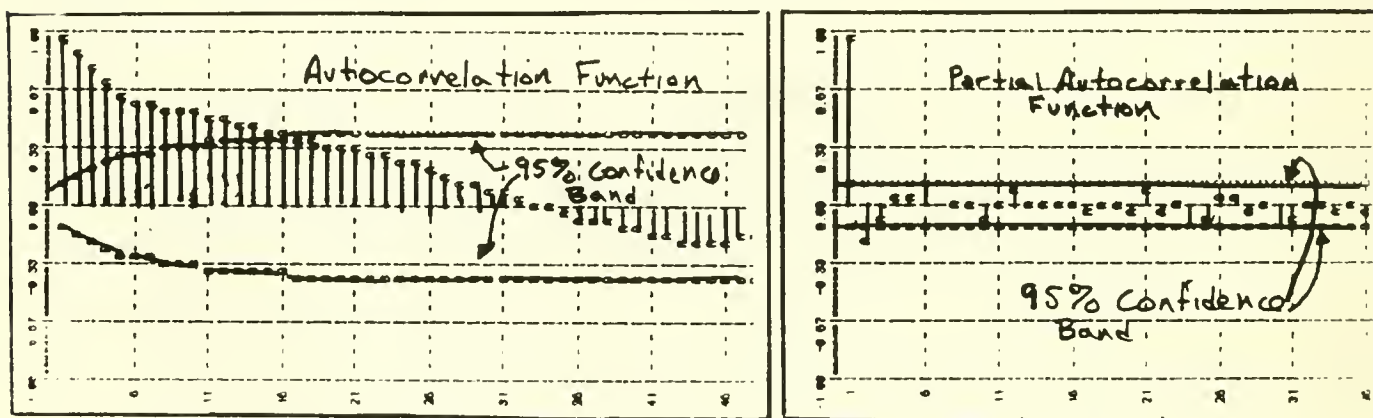
16. No intercept constant appears in 20 because such a constant would imply that balanced liquidity among depository institutions implies imbalanced liquidity elsewhere in the economy. This seems unreasonable.

17. Bias is also introduced by the error term in (20). In the absence of an obvious instrument no correction has been made for this factor.

18. The time period chosen was determined by the availability of time series data. A description of the data used may be found in appendix 1.

Figure 1: Estimation Results									
	k	z	h	ρ_1	ρ_2	r^2	DW	SSR	NOB
1 se	1.3703 .019701	- 4.95736 .409873	.024539 .001307			.713788	.100463	21.1888	318
2 se	1.48894 .158782	- 1.65294 .264725	.062574 .025783	1.23952 .054322	- .272328 .054134	.982018	1.96239	1.30528	316
3 se	1.42117 .207525	- 1.1465 .672131	.029146 .156981	1.24745 .078528	- .277475 .078321	.967317	1.93207	.531059	157
4 se	1.37445 7.16551	- 1.7628 .309902	.004235 .042859	1.24128 .077061	- .284951 .075898	.95874	1.93973	.752594	157
r^2 - corrected R-square; DW - Durbin Watson statistic ; SSR - Sum of squared residuals, NOB - Number of observations									

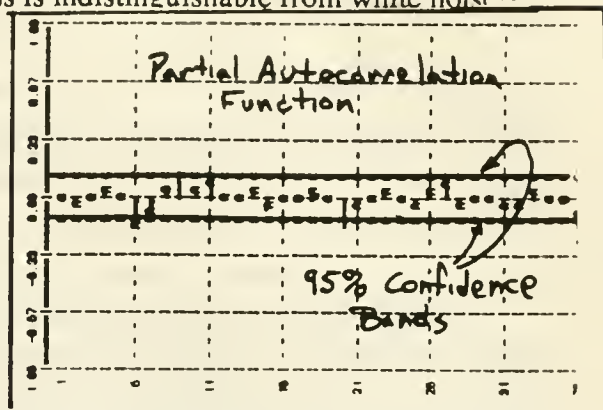
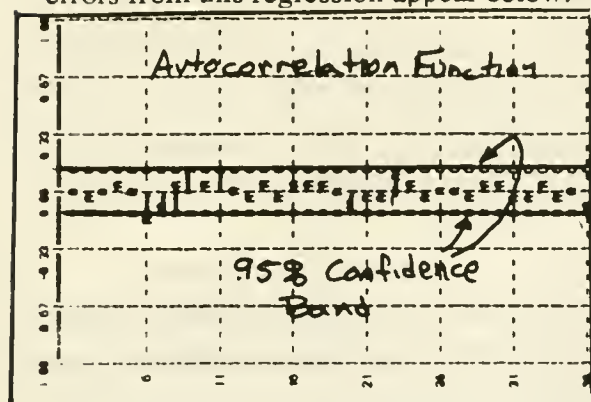
The low Durbin-Watson statistic in line 1 suggests the presence of an autocorrelated disturbance term. The autocorrelation and partial autocorrelation function for the errors appears below.



The gradual, but uneven, fall in the auto-correlation function and the two initial spikes in the autocorrelation function suggests that the errors may follow a second order autocorrelated process (Box and Jenkins 1976).

The structure of the errors prompts a reestimation using the Cochrane-Orcutt procedure to make a second order autocorrelation correction (M.I.T. Center for Computational Research 1983, p. 4-1-46 to 4-1-49; see also Johnston 1972, pp. 261-263) Results are summarized in line 2 in figure 1. The time constant (TAUIR) implied by the estimate for h is a reasonable 16 months or 1.3 years. The estimate $(-z)$ of α^*j is 1.65294. If α is between 1 and 2, j is between 1.65 and

.83—a reasonable range. The autocorrelation function and partial autocorrelation functions for the errors from this regression appear below. The process is indistinguishable from white noise¹⁹



In order to test the stability of these results the data has been divided in half and regressions performed on each half. The third line of figure 1 contains estimates using data from January of 1959 to March of 1972, while the fourth line contains estimates using data from March of 1972 to June of 1985. It is clear that decreasing the amount of data increases the variance of the estimates of both h ($1/TAUIR$) and z ($-\alpha*j$). Taking this increase in variance into account, the estimates appear to be within tolerable ranges of one another indicating that the coefficients are stable. The Chow test (Pindyck and Rubinfeld, p. 123-126) gives an explicit test of the hypothesis that the regressions are identical. The F-statistic for this case is 1.0221. Under the hypothesis that the regressions are identical, large values of the statistic are increasingly unlikely. Statistics greater than 1.0221 would be expected with a probability greater than one half. Consequently, the hypothesis seems to be quite acceptable. More formally, the hypothesis that the regressions are identical cannot be rejected at the fifty per cent confidence level.

4. Summary and Conclusions

In this paper a behavioral theory of interest rate formulation has been developed and tested. The theory postulates a disequilibrium adjustment process. At the heart of the theory is an intermediary willing to buy and sell for its own account at prices which do not immediately equate supply and demand.

The important informational input into the price setting formulation is the intermediary's inventory of money. The inventory of money is an accumulation of the history of the non-financial sectors' realized purchases and sales of securities. A low money inventory indicates that the non-financial sectors have been selling securities at a rate greater than they have been buying them. An increase in the interest rate is the appropriate response in this situation. Real-life intermediaries, in

19. As a check, a regression employing a first order correction was performed. The error terms were very definitely not white noise.

fact, are motivated to increase rates under these circumstances. The converse applies when money inventories are high and the intermediary is highly liquid.

The interest rate is adjusted up or down relative to an anchor termed the "underlying interest rate". The underlying interest rate is the rate to which people have become accustomed. It is represented mathematically as an weighted average of past values of the actual interest rate with recent experience weighted most heavily.

Empirical support for the formulation exists at two levels of aggregation. Research into the nature of human decision making at the individual level indicates that adjustment and anchoring is a common strategy for making decisions like those involved in interest rate determination. Econometric estimation was used to validate the process at a macroeconomic level of aggregation.

The research reported in this paper can be extended in two directions: one empirical and the other theoretic. Better data can be collected on the reserves and desired reserves of intermediaries. Alternatively, more sophisticated estimation techniques might be utilized to correct for biases introduced by using surrogate measures. Observation of the way interest rates are actually determined by those who actually determine them would provide an even more valuable opportunity to confirm or disconfirm the formulation suggested in this paper. Observation at the individual decision making level would also provide the opportunity to refine the theory presented here.

Theory might also be extended through disaggregation. While this paper dealt at a highly aggregated level, the disequilibrium process described herein could be applied to a more detailed model which included several different securities and several different intermediaries. Such an effort would involve a shift in purpose. The overarching goal of the current research has been a theory of interest rate formation realistic enough to contribute to, yet simple enough not to obfuscate, an understanding of the major macroeconomic behavior modes. The goal of disaggregation would be the increased understanding of behavior modes in the financial markets.

Appendix 1: Data Sources

This appendix provides information on the data, and their transformation, used to estimate equation 19. The risk-free rate is based on CITIBASE's (Citibank 1985) secondary market rate on three month treasury bills (data series FYGM3, see also Federal Reserve Bulletin, table 1.20)). This rate is calculated on a bank-discount basis. To convert this to a continuous time yield, the following calculation were performed. First, the percentage discount (DISC) is

$$\text{DISC} = \text{FYGM3} / 4.$$

A1.1

The decimal discrete-time yield (DTY) may now be written as

$$DTY = DISC / (100 - DISC). \quad A1.2$$

This may be converted to a continuous-time yield (CTY) as

$$CTY = \ln (1 + DTY). \quad A1.3$$

Finally, prior to performing regressions, this was converted to a yearly percentage basis:

$$RFIR = CTY * (365/90) * 100 \quad A1.4$$

Relative available reserves RAR are defined as the difference between reserves and desired reserves, divided by reserves. The data series used for reserves was CITIBASE's FZRNBA, nonborrowed reserves of depository institutions, and for desired reserves CITIBASE's FZRQA, required reserves of depository institutions. Neither FZRNBA nor FZRQA are seasonally adjusted. The series are, however, adjusted for changes in reserve requirements, in particular the change in institutions required to hold reserves associated with the Monetary Control Act of 1980.

The correlation between the RAR used in this report and an alternate RAR constructed with non-seasonally adjusted, non-reserve-adjusted time series (FZRNBA and FZRQA) is .976. The alternate definition of RAR was tested over the full range of observations (1959/1 to 1985/6). Parameter estimates were insignificantly different from those reported in the text using data adjusted for changes in reserve requirements.

Appendix 2: Modification to Interest Rate Structure

As discussed in the text, the structure contains a risk that rates will become "stuck" at zero. A small modification removes this risk. The change involves introducing a new variable, the indicated underlying rate IUR, toward which the underlying rate adjusts. The indicated underlying rate is identical to the risk-free rate, except the underlying rate does not go to zero, but rather to some minimum value.

Behaviorally such a modification implies there is some lowest reference rate which is greater than zero. Economically this minimum rate may be determined by the costs of intermediation. The minimum rate may be interpreted as the average cost of intermediation expressed as a fraction of the amount intermediated.

A restatement of the interest rate formulation using a possible definition of the indicated underlying rate is²⁰:

20. A function involving a gradual approach to MUIR, in place of the abrupt change introduced by the max function, would also be a possible assumption. The use of such a function, however, introduces additional complications and further dynamics without materially enriching the current discussion.

$$RFIR_t = UIR_t * ELR_t \quad (A2.1)$$

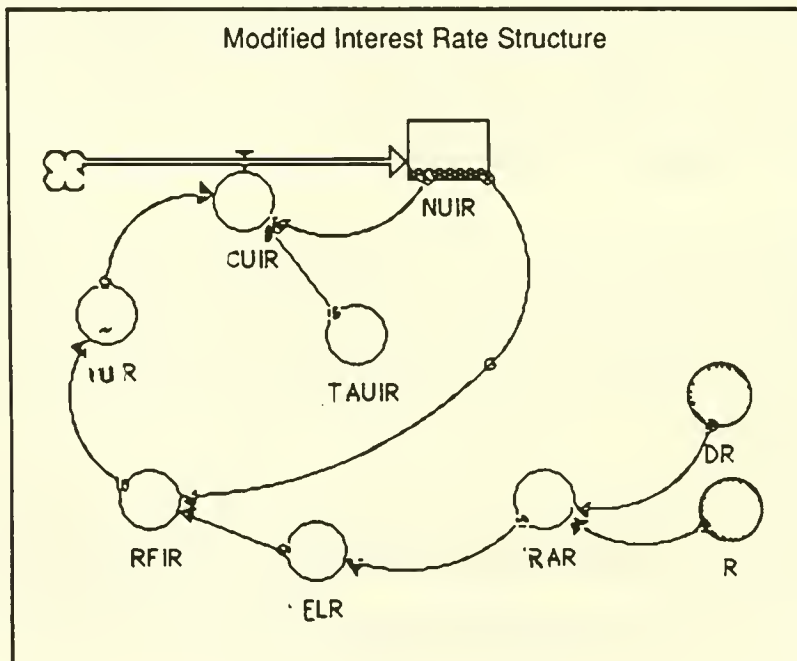
$$UIR_t = (IUR_t - UIR_t) / TAUIR \quad (A2.2)$$

$$IUR_t = \max(RFIR_t, MUIR) \quad (A2.3)$$

$$ELR_t = e^{-\alpha * RAR_t} \quad (A2.4)$$

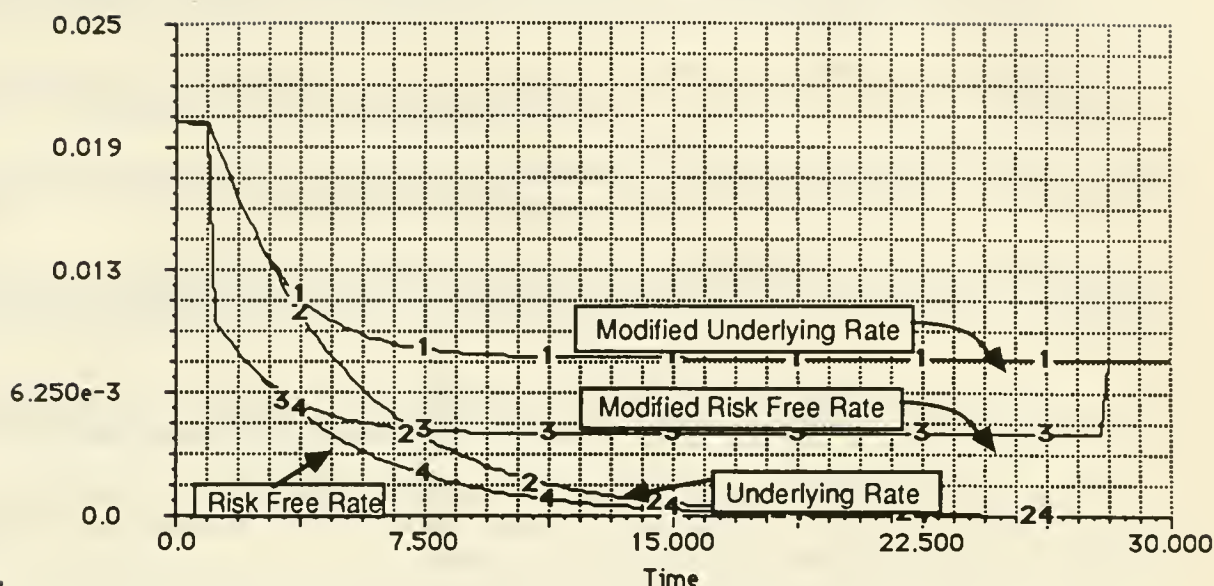
Where: RFIR	-Risk Free Interest Rate
UIR	-Underlying Risk Free Interest Rate
ELR	-Effect of Liquidity on the risk free Rate
IUR	- Indicated Underlying Interest Rate
MUIR	- Minimum Underlying Interest Rate, a constant
TAUIR	- Time to Adjust the Underlying Interest Rate
RAR	- Relative Available Reserves
e	- The exponential function
α	- a constant

The new definition of the underlying interest rate appears as A2.3. The modified stock and flow diagram appears below.



A comparison of the behavior between the original formulation and the modified formulation appears below. In the simulation, TAUIR and α are set to 2. The structure is disturbed from equilibrium by a fifty per cent step increase in reserves around year 1. Reserves step down to their original level around year 28.

Behavior of Modified vs. Un-Modified Structure



The diagram above illustrates the possibility that the (unmodified) Risk Free Rate and (unmodified) Underlying Rate can "stick" at zero. It shows that the modified structure eliminates this risk. It should be noted that although the structure keeps the underlying rate from dropping below MUIR, the risk free rate may drop lower.

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